the PiXL club
partners in excellence

Practice Paper

GCSE Mathematics (Edexcel style)
June 2018
Higher Tier
Paper 3H
WORKED SOLUTIONS

Name ....

Class ....

TIME ALLOWED
1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- **You are permitted to use a calculator in this paper.**
- Do all rough work in this book.

INFORMATION FOR CANDIDATES
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper**.**
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **80**.

Question	Mark	Out of
1		3
2		3
3		4
4		3
5		4
6		3
7		3
8		7
9		4
10		3
11		3
12		4
13		4
14		6
15		7
16		6
17		5
18		3
19		5
Total		80

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

Question 1.

(a) Use your calculator to work out

$$\frac{9.3^2 + \sqrt{98.05}}{0.253}$$

$\frac{96.39202}{0.253}$ M1

Write down all the digits on your calculator display.

380.9961265 A1

....

(2)

(b) Write your answer to part (a) correct to 2 significant figures.

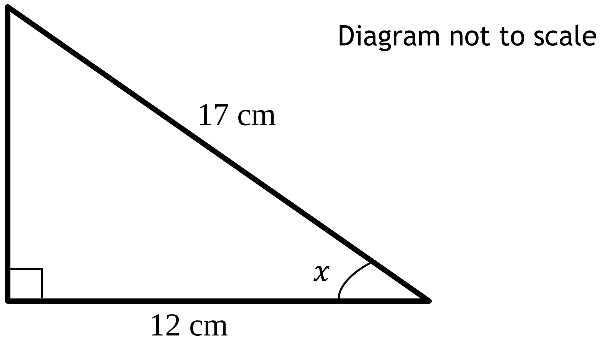
380 B1

....

(1)

(Total 3 marks)

Question 2.



Work out the value of *x*.
Give your answer correct to 2 significant figures.

$\cos x = \frac{12}{17}$ M1

$\cos^{-1} \frac{12}{17} = x$ M1

x = ...45 A1.......... °

(Total 3 marks)

Question 3.

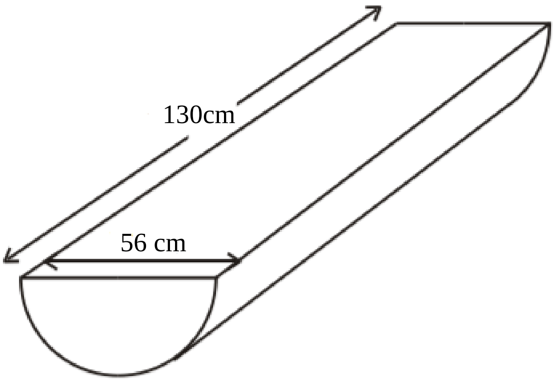


Diagram not to scale

The diagram shows a piece of wood.

The piece of wood is a prism of length 130 cm.

The cross-section of the prism is a semi-circle with diameter 56 cm.

Calculate the surface area of the piece of wood.

Give your answer correct to 1 decimal place.

Area of cylinder $2\pi r^2 + \pi d l$

$= (2\pi \times 28^2) + (\pi \times 56 \times 130)$

$= 1568\pi + 7280\pi$ P1

$= 8848\pi$

½ of cylinder

$= \frac{8848\pi}{2}$

$= 4424\pi$

$4424\pi + 7280$ P1

$= 21178.4059$

Area of rectangle

$56 \times 130 = 7280$ P1

A1 21178.4 ....cm^2

(Total 4 marks)

Question 4.

Work out the value of $\dfrac{5p+q^2}{2p}$, where $p = 4.7 \times 10^3$ and $q = 7.6 \times 10^3$.

Give your answer in standard form to 3 significant figures.

$$= \frac{5 \times (4.7 \times 10^3) + (7.6 \times 10^3)^2}{2 \times (4.7 \times 10^3)} \qquad \text{M1}$$

$$= \frac{57783500}{9400} \qquad \text{M1}$$

$$= 6147.1808\ldots$$

6.15×10^3 A1

……………………………………………..

(Total 3 marks)

Question 5.

Make a the subject of the formula

$$P = \frac{n^2 + a}{n + a}$$

$p(n + a) = n^2 + a$ **M1**

$pn + pa = n^2 + a$

$pa - a = n^2 - pn$ **M1**

$a(p - 1) = n^2 - pn$ **M1**

$a = \dfrac{n^2 - pn}{p - 1}$ **A1**

……………………………………………

(Total 4 marks)

Question 6.

Show that the recurring decimal $0.34\dot{7}$ can be written as $\frac{313}{900}$.

$x = 0.34\dot{7}$

$100x = 34.\dot{7}$ M1

$1000x = 347.\dot{7}$ M1

$900x = 313$

$x = \frac{313}{900}$ A1

....

(Total 3 marks)

Question 7.

Dan and Sam each have an expression.

Dan

$(x + 2)^2 - 36$

Sam

$(x + 8)\,(x - 4)$

Show clearly that Dan's expression is equivalent to Sam's expression.

P1

$(x+2)(x+2) - 36$

$x^2 + 4x + 4 - 36$

$x^2 + 4x - 32$ A1

$(x+8)(x-4)$

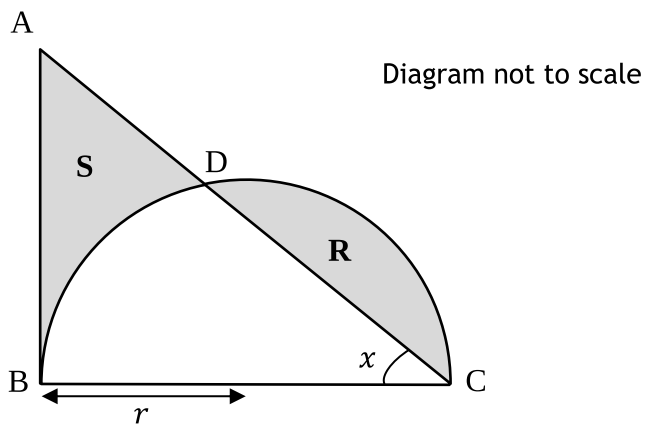
M1 $x^2 + 8x - 4x - 32$

$x^2 + 4x - 32$

(Total 3 marks)

Question 8.

The diagram shows a semi-circle and a triangle.



BC is a diameter of a semi-circle.

Angle ABC = 90°.

Area of S = Area of R.

Angle ACB = x.

The radius of the semi-circle is r.

(a) Find the length of AB in terms of r and x.

$\tan x = \frac{AB}{2r}$ M1

$AB = 2r\tan x$

$2r\tan x$ A1 ....

(2)

(b) Show that $\tan x = \frac{\pi}{4}$.

Area of Δ $\frac{2r\tan x \times 2r}{2}$ M1

Area of semi-circle $\frac{\pi r^2}{2}$

$\frac{2r\tan x \times 2r}{2} = \frac{\pi r^2}{2}$ M1

$8r^2\tan x = 2\pi r^2$

$\tan x = \frac{2\pi r^2}{8r^2}$ A1

$\tan x = \frac{\pi}{4}$ ....

(3)

(c) Find angle x to 1 decimal place.

$\tan^{-1}\frac{\pi}{4} = x$ M1

$x = 38.1$ A1 ....°

(2)

(Total 7 marks)

Question 9.

E and F are points on the circumference of a circle with centre O.

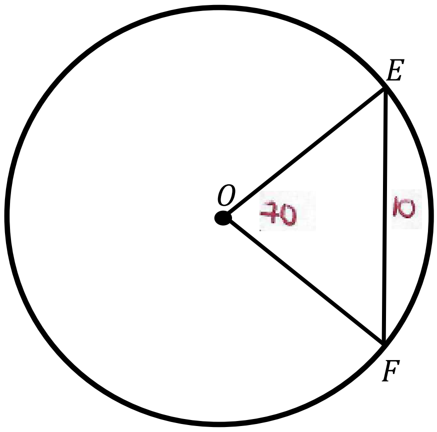


Diagram not to scale

EF = 10 cm and angle EOF = 70°.

Calculate the circumference of the circle.

Give your answer to 3 significant figures.

OE 35 5

$\sin 35 = \frac{5}{OE}$ P1

$OE = \frac{5}{\sin 35}$

$OE = 8.717233978$ M1

$C = \pi d$

$C = \pi \times (8.717\ldots \times 2)$ M1

$C = 54.77199645$

A1 54.8 ....cm

(Total 4 marks)

Question 10.

A full-size snooker ball has a diameter of $2\frac{1}{16}$ inches and weighs 302.5g.

Calculate the weight of a snooker ball of diameter $1\frac{7}{8}$ inches, assuming that both balls are made of the same material.

Give your answer to the nearest gram.

$\frac{33}{16} \div \frac{15}{8}$

$\frac{33}{16} \times \frac{8}{15} = \frac{264}{240} = \frac{11}{10}$

LSF $= \frac{11}{10}$ M1

VSF $= \frac{1331}{1000}$

$302.5 \div \frac{1331}{1000}$ M1

$= 227.\dot{2}\dot{7}$

A1 227 ……………………………………………g

(Total 3 marks)

Question 11.

Given that *x* = 3.2 correct to 1 decimal place, find the interval that contains the value of $5x^2 + 4$.

Give your answer as an inequality.

$0.1 \div 2 = 0.05$

UB 3.25
LB 3.15 P1

$(5 \times 3.25^2) + 4 = 56.8125$ M1

$(5 \times 3.15^2) + 4 = 53.6125$

$53.6125 \leq 5x^2 + 4 < 56.8125$ A1 ……………………………………………

(Total 3 marks)

Question 12.

The speed and acceleration of a moving vehicle are connected by the formula $v^2 = u^2 + 2as$.

If $u = 4\sqrt{3}$, $a = \sqrt{2}$ and $s = 7\sqrt{2}$,

Find the value of v.

Give your answer in surd format.

$v^2 = (4\sqrt{3})^2 + (2 \times \sqrt{2} \times 7\sqrt{2})$ P1

$= 48 + 28$ P1

$= 76$

$v = \sqrt{76}$ P1

$v = 2\sqrt{19}$

$v =$ .... $2\sqrt{19}$ A1 ....

(Total 4 marks)

Question 13.

12 grams of pond weed was introduced into a pond.

The weight of the weed in the pond 3 days later was 96g.

The weight of the weed in the pond is growing exponentially.

Work out the weight of the weed in the pond after 8 days.

$12 \times x^3 = 96$ M1

$x^3 = 96 \div 12$

$x^3 = 8$

$x = \sqrt[3]{8}$

$x = 2$ M1

After 8 days:

12×2^8 M1

3072g A1

(Total 4 marks)

Question 14.

The sides of a triangle ABC are tangents to a circle.

The tangents touch the circle at the points D, E and F.

$BD = 8$ cm. $AD = 9$ cm.

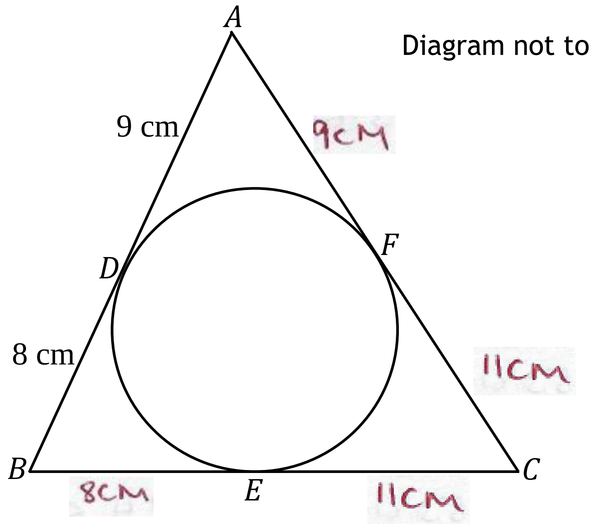


Diagram not to scale

(a) (i) Write down the length of BE.

8 B1 ....cm

(1)

(ii) Give a reason for your answer.

Tangents from the same point are equal. C1 ....

(1)

The perimeter of the triangle ABC is 56 cm.

(b) Calculate the size of the angle ABC.

Give your answer correct to 1 decimal place.

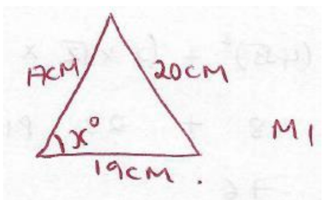
$56 - 17 - 17 = 22$

$22 \div 2 = 11$

$\cos x = \dfrac{17^2 + 19^2 - 20^2}{2 \times 17 \times 19}$ M1

$\cos x = \dfrac{250}{646}$ M1

$\cos^{-1} \dfrac{250}{646} = x$



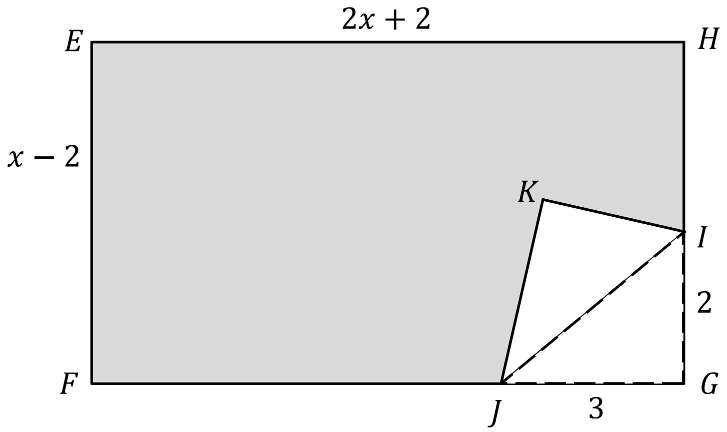
$x = 67.2^{\circ}$ A1 ....°

(4)

(Total 6 marks)

Question 15.

The diagram below shows a rectangular piece of paper $EFGH$.



The rectangle has been folded along IJ so that G moved to K.

(a) Find an expression for the shaded area in terms of x.

Area of rectangle $(2x+2)(x-2)$

$2x^2+2x-4x-4$ M1

$2x^2-2x-4$

Area of Δ $\frac{3\times 2}{2} = 3$ M1

$2x^2-2x-4-3-3$

$2x^2-2x-10$ A1 ....

(3)

(b) Given that the shaded area is 10 cm^2, prove that $x = \sqrt{x+10}$

$2x^2-2x-10 = 10$ M1

$2x^2-2x-20 = 0$

$x^2-x-10 = 0$

$x^2 = x+10$

$x = \sqrt{x+10}$ A1

....

(2)

(c) Use an iterative method to find the value of x to 2 significant figures.

Take $x_0 = 3$

$x_1 = \sqrt{3+10} = 3.6055\ldots$ or $\sqrt{13}$

$x_2 = \sqrt{\sqrt{13}+10} = 3.688570357$ M1

$x_3 = \sqrt{3.6885..+10} = 3.6998068$

$x =$ ......3.7 A1......

(2)

(Total 7 marks)

Question 16.

A curve has equation $y = 4x^2 - 8x + 25$.

(a) Write the expression $4x^2 - 8x + 25$ in the form $a(x + b)^2 + c$.

$4\left(x^2 - 2x + \frac{25}{4}\right)$ M1

$4\left[(x-1)^2 - 1^2 + \frac{25}{4}\right]$ M1

$4(x-1)^2 + 21$ M1

$a = 4$

$b = -1$ A1

$c = 21$

……………………………………………………

(4)

(b) Find the coordinates of the minimum point of the graph.

$(1, 21)$ B1

(1)

(c) State if and where the graph of the equation crosses the x-axis.

Minimum value of y is 21, ∴ the graph will always be above 0 on the y-axis. The curve will not cross the x-axis. C1

(1)

(Total 6 marks)

Question 17.

Find the equation of the tangent to $(x+1)^2 + (y+2)^2 = 169$ at the point (5, -12).

Centre $(-1, -2)$

gradient $\frac{-12 - -2}{5 - -1} = \frac{-10}{6} = \frac{-5}{3}$ P1

gradient of tangent is perpendicular, hence product of two gradients is '-1'.

Hence $\frac{3}{5}$ M1

$y = \frac{3}{5}x + c$ $(5, -12)$

$-12 = \frac{3}{5} \times 5 + c$ M1

$-12 = \frac{15}{5} + c$

$-15 = c$ M1

$y = \frac{3}{5}x - 15$ A1

....

(Total 5 marks)

Question 18.

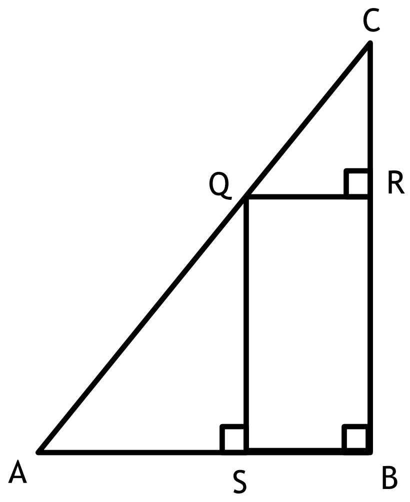


Diagram not to scale

ABC is a triangle with angle ABC = 90°.

S, Q and R are points on AB, AC and CB respectively such that angle QRC = angle ASQ = 90°.

Prove that triangle CQR is similar to triangle QAS.

Give a reason for each step of your proof.

QRC = angle ASQ = 90°   M1

CQR = QAS corresponding angles

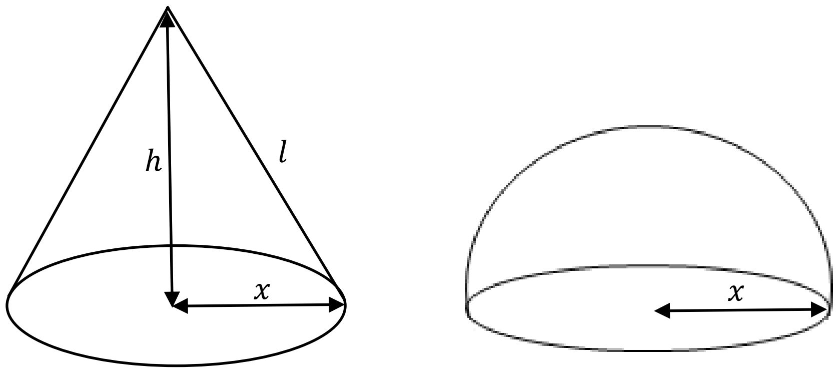
Or AQS = QCR corresponding angles   M1

Same angles, so similar triangles   C1

(Total 3 marks)

Question 19.

The diagram below shows a solid cone and a solid hemisphere.



Surface area of sphere is $4\pi r^2$
Curved surface area of a cone is $\pi r l$

The surface area of the cone is equal to the surface area of the hemisphere.

Express h in terms of x.

$l = \sqrt{x^2 + h^2}$ P1

$\pi x^2 + \pi x\sqrt{x^2+h^2} = \pi x^2 + 2\pi x^2$ M1

$\pi x\sqrt{x^2+h^2} = 2\pi x^2$ M1

$\sqrt{x^2+h^2} = 2x$

$x^2 + h^2 = 4x^2$ M1

$h^2 = 3x^2$

$h = \sqrt{3}\,x$

$h =$ $\sqrt{3}\,x$ A1 ....

(Total 5 marks)

TOTAL FOR PAPER IS 80 MARKS